**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:

Mean = 55

Standard Deviation = 8

Time Taken = 60

p = 1 - stats.norm.cdf(60,55,8)

The probability that the service manager cannot meet his commitment is 0.26599

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:

Mean = 38

Standard Deviation = 6

Employees = 400

Employees under 38 = stats.norm.cdf(38,m,std) = 0.5

Employees under 44 = stats.norm.cdf(44,m,std) = 0.84

Employees between 38 & 44 = 0.5 – 0.84 = 34.13%

Employees above 44 = 1 – 0.84 = 0.158

There are more employees at the processing center that are between 38 and 44 than that are above 44. Hence, it is a False Statement.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

Employees under 30 = stats.norm.cdf(30,m,std) = 0.09

No. of Employees under 30 = 0.09 \* no.of employees

= 0.09\*400 = 36 Employees

Hence, the Statement above is True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: According to the Central Limit Theorem, any large sum of independent, independent identically distribution random variables is normal.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 ) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then (X1 + X2)N(μ+ μ, σ2+ σ2)(2 μ, 2 σ2) Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 ) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters. The difference between the two says that the two given variables are identically and independently distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

Mean = 100

Standard Deviation = 20

Confidence Level = 0.99

= stats.norm.interval(cof,m,std)

= [ 48.5 151.5]

The two values of a and b, symmetric about the mean, are such that the probability of the random variable taking a value between them is 0.99 is [ 48.5 151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Ans :

Mean 1 = 5

Mean 2 = 7

Total mean = 5+7 = 12

Standard Deviation 1 = 3\*\*2

Standard Deviation 2 = 4\*\*2

Standard Deviation = np.sqrt(std1+std2) = 5

Mean in Dollars = 12 Million Dollars

Mean in rupees = (12\*45)/10 = 54 Crore Rupees

Standard Deviation in Dollars = 5 Million Dollars

Standard Deviation in rupees = (5\*45)/10 = 22.5 Crore Rupees

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

stats.norm.interval(0.95,m,std) = [2.2-21.8]

The range such that it contains 95% probability for the annual profit of the company is 2.2 Million Dollars to 21.8 Million Dollars

The range such that it contains 95% probability for the annual profit of the company is 9.9 Crore Rupees to 98.1 Crore Rupees

1. Specify the 5th percentile of profit (in Rupees) for the company

stats.norm.ppf(0.05,m,std) = 3.78

The 5th percentile of Profit for the company is 3.78 Million Dollars

The 5th percentile of Profit for the company is 16.99 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Division 1 = stats.norm.cdf(0,5,3) = 0.048

Division 2 =stats.norm.cdf(0,7,4) = 0.04

The Division 1 has a larger Probability of making a loss.